

# Pion dominance in $\tilde{R}_p$ SUSY induced neutrinoless double beta decay

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At the quark level there are basically two types of contributions of R-parity violating SUSY ( $\tilde{R}_p$  SUSY) to neutrinoless double beta decay: the short-range contribution involving only heavy virtual superpartners and the long-range one with the virtual squark and neutrino. Hadronization of the effective operators, corresponding to these two types of contributions, may in general involve virtual pions in addition to close on-mass-shell nucleons. From the previous studies it is known that the short-range contribution is dominated by the pion exchange. In the present paper we show that this is also true for the long-range  $\tilde{R}_p$  SUSY contribution. Therefore, we conclude that the  $\tilde{R}_p$  SUSY contributes to the neutrinoless double beta decay dominantly via charged pion exchange between the decaying nucleons.

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## I. INTRODUCTION

The nuclear neutrinoless double beta decay ( $0\nu\beta\beta$ ) is a process known for 70 years, which has been searched for, but not yet seen. The  $0\nu\beta\beta$ -decay plays a special role among exotic processes for the following two reasons. First, it is intimately related to the nature of the neutrino[1], since it is able to probe whether the neutrino is a Dirac or a Majorana particle. Second, the modern  $0\nu\beta\beta$ -experiments have reached unprecedented sensitivity. The most stringent lower bound on the half-life of the  $0\nu\beta\beta$ -decay was measured for  $^{76}\text{Ge}$  in the Heidelberg-Moscow experiment [2]:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 1.9 \times 10^{25} \text{ yrs.} \quad (1)$$

In the near future this limit is expected to be improved in the GERDA experiment [3] by 1-2 orders of magnitude. For the  $0\nu\beta\beta$ -decays of  $^{100}\text{Mo}$  and  $^{130}\text{Te}$  in the two running experiments, NEMO3 [4] and CUORICINO [5], the following sensitivities have been achieved:

$$\begin{aligned} T_{1/2}^{0\nu}(^{100}\text{Mo}) &\geq 5.8 \cdot 10^{23} \text{ years} \\ T_{1/2}^{0\nu}(^{130}\text{Te}) &\geq 3.0 \cdot 10^{24} \text{ years.} \end{aligned} \quad (2)$$

The extraordinary sensitivity of  $0\nu\beta\beta$ -experiments makes them also a unique laboratory tool to probe physics beyond the standard model (SM) with possible lepton number violation (LNV) underlying. Of particular interest in this context are supersymmetric models with R-parity violation ( $\tilde{R}_p$  SUSY) containing the LNV interactions necessary to generate the Majorana mass for neutrinos and to induce the  $0\nu\beta\beta$ -decay. There is a wealth of literature on the  $\tilde{R}_p$  SUSY mechanisms of

the  $0\nu\beta\beta$ -decay [6, 7, 8, 9, 10, 11, 12, 13, 14, 15] where the corresponding quark-level LNV interactions as well as hadronic and nuclear structure aspects relevant for these mechanisms are studied. At the quark-level there are basically two types of  $\tilde{R}_p$  SUSY mechanisms: the short-range mechanism with the exchange of heavy superpartners [6, 7, 8, 9, 10, 11] and the long-range mechanism involving both the exchange of heavy squarks and the light neutrino [12, 13, 14, 15], which we call squark-neutrino mechanism. For the latter case, due to the chiral structure of the  $\tilde{R}_p$  SUSY interactions, the amplitude of  $0\nu\beta\beta$ -decay does not vanish in the limit of zero neutrino mass in contrast to from the ordinary Majorana neutrino exchange mechanism proportional to the light neutrino mass. Instead, the squark-neutrino mechanism is roughly proportional to the momentum of the virtual neutrino, which is of the order of the Fermi momentum of the nucleons inside the nucleus with  $p_F \approx 100\text{MeV}$ . This is a manifestation of the fact that the LNV necessary for  $0\nu\beta\beta$ -decay is supplied by the  $\tilde{R}_p$  SUSY interactions instead of the Majorana neutrino mass term and, therefore, this mechanism is not suppressed by the small neutrino mass.

In the calculation of the  $0\nu\beta\beta$ -decay amplitude one has to hadronize the quark-level LNV operators representing them in terms of the interpolating nucleon fields. As is known [9], this can be done in two ways: the quark fields can be directly imbedded into the interpolating two nucleon fields (the two nucleon mode) or via an intermediate step when one quark-antiquark pair is associated with the charged pion field coupled to the nucleons (pion mode). In Refs. [9, 11] it was shown that the pion mode of hadronization dominates over the two nucleon one for the case of the short-range mechanism. In the present paper we demonstrate that the same conclusion is valid for the long-range squark-neutrino mechanism. Therefore, in all cases the dominant contribution of the  $\tilde{R}_p$  SUSY to the  $0\nu\beta\beta$ -decay is realized via the pion mode of hadroniza-

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tion. For the calculation of the nuclear matrix elements (NMEs) of the corresponding transition operators we apply the proton-neutron Quasiparticle Random Phase Approximation (pn-QRPA) [16].

The paper is organized as follows. In Sect. II we introduce the quark-level process related to the squark-neutrino mechanism of the  $0\nu\beta\beta$ . In Sect. III we discuss the two nucleon and the pion modes of hadronization of the effective quark-level Lagrangian. The  $0\nu\beta\beta$  NMEs of the corresponding transition operators are presented in Sect. IV. For the three nuclei of experimental interest the values of these NMEs are calculated in the QRPA. They are discussed in Sect. V. We summarize and discuss our results in Section VI.

## II. SQUARK-NEUTRINO MECHANISM OF $0\nu\beta\beta$ -DECAY

Lepton number violation in the fermions sector is generated within  $\tilde{R}_p$  SUSY models by the following terms of the superpotential:

$$W_{\tilde{R}_p} = \frac{1}{2}\lambda_{ijk}L_iL_j\bar{E}_k + \lambda'_{ijk}L_iQ_j\bar{D}_k + \kappa^iL_iH_2, \quad (3)$$

where  $L$  and  $Q$  are the left-handed lepton and quark superfield  $SU(2)$  doublets, while  $\bar{E}$ ,  $\bar{U}$  and  $\bar{D}$  denote the right-handed lepton, up-quark and down-quark  $SU(2)$  singlets, respectively.

The mixing between the scalar superpartners  $\tilde{q}_{L,R}$  of the left and right-handed quarks  $q_{L,R}$  plays the crucial role for the case of the long-range  $\tilde{R}_p$  SUSY squark-neutrino (SQN) mechanism. This effect occurs due to the non-diagonality of the squark mass matrix. For the down squarks of each generation it takes the form (e.g. [17])

$$\mathcal{M}_{\tilde{d}}^2 = \begin{pmatrix} m_{\tilde{d}_L}^2 + m_d^2 - 0.42D_Z & -m_d(A_d + \mu \tan\beta) \\ -m_d(A_d + \mu \tan\beta) & m_{\tilde{d}_R}^2 + m_d^2 - 0.08D_Z \end{pmatrix}. \quad (4)$$

Here,  $d = d, s, b$  and  $\tilde{d}$  are their superpartners.  $D_Z = M_Z^2 \cos 2\beta$  where  $\tan\beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$  is the ratio of vacuum expectation values of the two Higgs doublets,  $m_{\tilde{d}_{L,R}}$  are the soft squark masses,  $A_d$  are the soft SUSY breaking parameters describing the strength of the trilinear scalar interactions, and  $\mu$  is the supersymmetric Higgs(ino) mass parameter. Once squark mixing is included, the current eigenstates  $\tilde{d}_L, \tilde{d}_R$  become superpositions of the mass eigenstates  $\tilde{d}_i$  with the masses  $m_{\tilde{d}_i}$  and the corresponding mixing angle  $\theta^d$  defined as

$$m_{\tilde{d}_{1,2}}^2 = \frac{1}{2} \left[ m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4m_{LR}^4} \right], \quad (5)$$

$$\sin 2\theta^d = \frac{2m_{(d)LR}^2}{m_{\tilde{d}_1}^2 - m_{\tilde{d}_2}^2}, \quad (6)$$

where  $m_{LR}^2, m_{LL}^2, m_{RR}^2$  denote the  $(1, 2), (1, 1), (2, 2)$  entries of the mass matrix (4).

The squark-neutrino mechanism is a long-range mechanism involving both heavy squark and light neutrino virtual states as shown in Fig. 1(a). The bottom part of the diagram corresponds to the ordinary SM charged current interaction. The  $\tilde{q}_L - \tilde{q}_R$  - mixing results in lepton number violation with  $\Delta L = 2$  through the  $\tilde{R}_p$  interactions proportional to the  $\lambda'\lambda'$ -coupling. Without the  $\tilde{q}_L - \tilde{q}_R$  - mixing the  $\tilde{R}_p$ -contribution would be proportional to the  $\lambda'\lambda'^*$ -coupling, conserving lepton number. Thus, in the latter case LNV with  $\Delta L = 2$  is introduced by the Majorana mass of the neutrino with the amplitude of  $0\nu\beta\beta$ -decay proportional to the small neutrino mass and, therefore, is strongly suppressed.

It is straightforward to derive the effective 4-fermion  $\nu - u - d - e$  vertex induced by the squark exchange corresponding to the top part of the diagram in Fig. 1(a). The corresponding effective Lagrangian, after a Fierz rearrangement, takes the form

$$\mathcal{L}_{SUSY}^{eff}(x) = \frac{G_F}{8\sqrt{2}} \eta_{(q)LR}^{n1} U_{ni}^* \times \\ [(\bar{\nu}_i \sigma^{\mu\nu} (1 + \gamma_5) e^c) (\bar{u} \sigma_{\mu\nu} (1 + \gamma_5) d) \\ + 2 (\bar{\nu}_i (1 + \gamma_5) e^c) (\bar{u} (1 + \gamma_5) d)], \quad (7)$$

with  $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$ . Here  $U$  is the neutrino mixing matrix and the SUSY LNV parameter is defined as

$$\eta_{(q)LR}^{nj} = \sum_k \frac{\lambda'_{j1k} \lambda'_{nk1}}{2\sqrt{2}G_F} \sin 2\theta_{(k)}^d \left( \frac{1}{m_{\tilde{d}_1(k)}^2} - \frac{1}{m_{\tilde{d}_2(k)}^2} \right). \quad (8)$$

Here we use the notations  $d_{(k)} = d, s, b$ . This LNV parameter vanishes in the absence of  $\tilde{q}_L - \tilde{q}_R$  - mixing when  $\theta^d = 0$ , in accordance with the previous comments.

## III. HADRONIZATION PRESCRIPTIONS

To evaluate the contribution of the diagram in Fig. 1(a) to  $0\nu\beta\beta$ -decay one first should express the quark fields in terms of nucleon ones. This procedure, known as hadronization, has so far a rather poor theoretical background. In practice the transformation is carried out in a phenomenological way, where different possibilities of embedding the quark fields into the interpolating hadronic fields are considered.

There are the two possibilities [9] of hadronization for the diagram of Fig. 1(a).

1. The four quark fields can be embedded in the two initial neutrons and two final protons separately [6, 7, 8]. This is the conventional two-nucleon (2N) mode of  $0\nu\beta\beta$ -decay shown in Fig. 2(b).

2. Another possibility is to incorporate one pair of (anti-)quarks of the underlying quark-level  $\tilde{R}_p$  SUSY transition into one virtual pion [9, 10, 11], while another (anti-)quark pair participates in the charged current SM

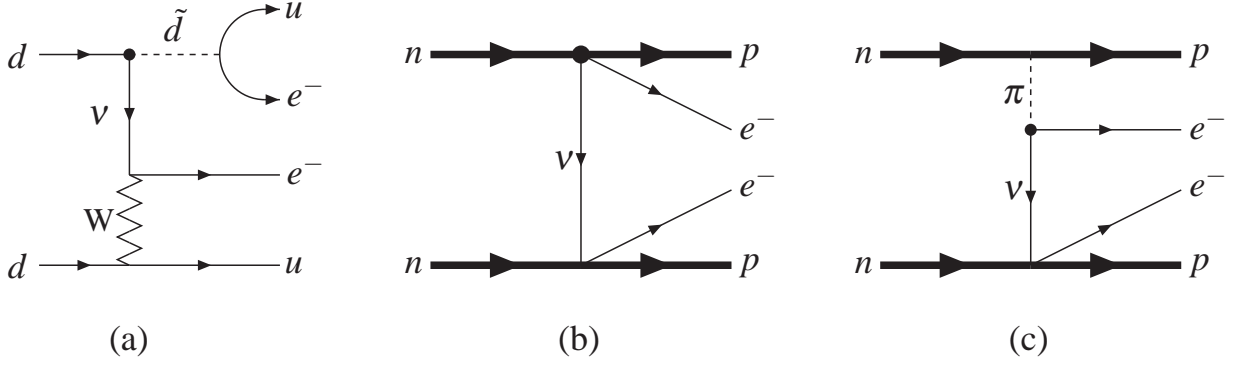


FIG. 1: Diagrams describing the squark-neutrino mechanism of  $0\nu\beta\beta$ -decay (a) at the quark-level; at the nucleon level with hadronization in the (b) 2N-mode and (c) the pion-mode. The dark blobs denote the LNV vertices generated by  $\tilde{R}_p$  SUSY trilinear interactions.

transition involving initial neutron and final proton. In this case, which we call the pion mode of  $0\nu\beta\beta$ -decay, the second (the upper one in Fig. 1(c)) nucleon vertex is connected to the leptonic LNV vertex through the charged pion exchange. Note that the pion exchange contribution in the charged current SM interaction (the bottom vertex in Fig. 1(c)) is automatically taken into account by the induced pseudoscalar form factor of the nucleon.

The effective hadronic Lagrangian describing the vertices of the diagrams in Figs. 1(b,c) can be derived using the method of Ref. [9], matching the hadronic matrix elements of the hadronic and the corresponding quark operators. This Lagrangian, taking into account both the nucleon (p, n) and  $\pi$ -meson degrees of freedom in a nucleus, has the following form:

$$\mathcal{L}_h = \mathcal{L}_\beta + \mathcal{L}_{R_p}^{PS} + \mathcal{L}_{R_p}^{PT} + \mathcal{L}_{R_p}^\pi + \mathcal{L}_s \quad (9)$$

with

$$\mathcal{L}_\beta = \frac{G_F}{\sqrt{2}} J_L^{\mu\dagger} \sum_i U_{ei} \bar{e} \gamma_\mu (1 - \gamma_5) \nu_i, \quad (10)$$

$$\mathcal{L}_{R_p}^{PS} = \frac{G_F}{4\sqrt{2}} \eta_{(q)LR}^{11} J_{PS} \sum_i U_{ei}^* \bar{\nu}_i (1 + \gamma_5) e^c, \quad (11)$$

$$\mathcal{L}_{R_p}^{PT} = \frac{G_F}{8\sqrt{2}} \eta_{(q)LR}^{11} J_{PT}^{\mu\nu} \sum_i U_{ei}^* \bar{\nu}_i \sigma_{\mu\nu} (1 + \gamma_5) e^c, \quad (12)$$

$$\mathcal{L}_{R_p}^\pi = i \frac{G_F}{4\sqrt{2}} \eta_{(q)LR}^{11} m_\pi^2 h_\pi \pi^- \sum_i U_{ei}^* \bar{\nu}_i (1 + \gamma_5) e^c \quad (13)$$

$$\mathcal{L}_s = g_{\pi NN} \bar{p} i \gamma_5 n \pi^+. \quad (14)$$

Here,  $\mathcal{L}_\beta$  is the charged current SM term,  $\mathcal{L}_{R_p}^{PS}$ ,  $\mathcal{L}_{R_p}^{PT}$  and  $\mathcal{L}_{R_p}^\pi$  are the  $\tilde{R}_p$  SUSY induced nucleon-electron-neutrino and pion-electron-neutrino interactions, respectively. The coefficient  $a_\pi$  in Eq. (13) is determined by the matrix element [9]

$$\langle 0 | \bar{u} \gamma_5 d | \pi^- \rangle = i \sqrt{2} f_\pi \frac{m_\pi^2}{m_u + m_d} \equiv i m_\pi^2 h_\pi, \quad (15)$$

where  $f_\pi = 0.668 m_\pi$ .

The Lagrangian term  $\mathcal{L}_s$  in Eq. (14) stands for the standard pion-nucleon interaction with the coupling  $g_{\pi NN} = 13.4 \pm 1$  known from experiment.

The nucleon currents are defined as [18]

$$\begin{aligned} J_L^{\mu\dagger} &= \langle P(p) | \bar{u} \gamma^\mu (1 - \gamma_5) d | N(p') \rangle \\ &= \bar{p} \left[ g_V(q^2) \gamma^\mu + i g_M(q^2) \frac{\sigma^{\mu\nu}}{2m_p} q_\nu \right. \\ &\quad \left. - g_A(q^2) \gamma^\mu \gamma_5 - g_P(q^2) q^\mu \gamma_5 \right] n, \end{aligned} \quad (16)$$

$$\begin{aligned} J_{PS} &= \langle P(p) | \bar{u} (1 + \gamma_5) d | N(p') \rangle \\ &= \bar{p} \left[ F_S^{(3)}(q^2) + F_P^{(3)}(q^2) \gamma_5 \right] n, \end{aligned} \quad (17)$$

$$\begin{aligned} J_{PT}^{\mu\nu} &= \langle P(p) | \bar{u} \sigma^{\mu\nu} (1 + \gamma_5) d | N(p') \rangle \\ &= \bar{p} \left( J^{\mu\nu} + \frac{i}{2} \epsilon^{\mu\nu\rho\sigma} J_{\rho\sigma} \right) n, \end{aligned} \quad (18)$$

where  $m_p$  is the nucleon mass,  $q_\mu = (p - p')_\mu$  is the momentum transferred to the nucleon vertex with  $p$  and  $p'$  being the four momenta of neutron and proton, respectively. The tensor structure is given by

$$\begin{aligned} J^{\mu\nu} &= T_1^{(3)}(q^2) \sigma^{\mu\nu} + \frac{i T_2^{(3)}(q^2)}{m_p} (\gamma^\mu q^\nu - \\ &\quad \gamma^\nu q^\mu) + \frac{T_3^{(3)}(q^2)}{m_p^2} (\sigma^{\mu\rho} q_\rho q^\nu - \sigma^{\nu\rho} q_\rho q^\mu). \end{aligned} \quad (19)$$

The nucleon form factors  $g_V(q^2)$ ,  $g_M(q^2)$ ,  $g_A(q^2)$ ,  $g_P(q^2)$   $F_S^{(3)}$ ,  $F_P^{(3)}$ ,  $T_k^{(3)}$  are real functions of the squared momentum  $q^2$  transferred to a nucleon. For all the form factors we use the following dipole parameterizations:

$$\frac{g_{V,A,M}(q^2)}{g_{V,A,M}} = \frac{F_{S,P}^{(3)}(q^2)}{F_{S,P}^{(3)}(0)} = \frac{T_i^{(3)}(q^2)}{T_i^{(3)}(0)} = \left( 1 - \frac{q^2}{\Lambda_V^2} \right)^{-2} \quad (20)$$

with  $\Lambda_V^2 = 0.71 (GeV)^2$  and the normalization constants:  $g_V = 1$ ,  $g_A = 1.254$ ,  $g_M = (\mu_p - \mu_n) g_V$ ,  $(\mu_p - \mu_n) = 3.70$ . The induced pseudoscalar coupling is given by the PCAC relation

$$g_P(q^2) = 2m_p g_A(q^2) / (m_\pi^2 - q^2). \quad (21)$$

TABLE I: Normalizations of nucleon form factors at  $q^2 = 0$  calculated in the quark bag model (QBM) and the non-relativistic quark model (NRQM). The table is taken from Ref. [18].

Set	$F_S^{(3)}$	$F_P^{(3)}$	$T_1^{(3)}$	$T_2^{(3)}$	$T_3^{(3)}$
QBM	0.48	4.41	1.38	-3.30	-0.62
NRQM	0.62	4.65	1.45	-1.48	-0.66

For the normalization constants  $F_{S,P}^{(3)}(0)$ ,  $T_i^{(3)}(0)$  we use the results of Ref. [18] summarized in Table I, obtained within the quark bag model and the non-relativistic quark model.

#### IV. $0\nu\beta\beta$ HALF-LIFE AND NUCLEAR MATRIX ELEMENTS

The matrix element of the SQN-mechanism (SQuark-Neutrino-mechanism) can be calculated according to the diagrams of Fig. 1(b,c) with the vertices described by the effective Lagrangian of Eq. (9). The vertices denoted by black blobs originate in squark exchange and correspond to the terms (11)-(13). The bottom parts of these diagrams are the standard model charged current interactions (10). Applying the standard procedure based on the non-relativistic impulse approximation (for details see [19]), it is straightforward to derive the nuclear matrix elements and the corresponding half-life formula for the  $0\nu\beta\beta$ -decay in the  $0^+ \rightarrow 0^+$  transition channel with two outgoing electrons in S-wave states. It takes the form:

$$\frac{1}{T_{1/2}} = G_{01} |M_h^{\tilde{q}}|^2 |\eta_{(q)LR}^{11}|^2, \quad (22)$$

where  $G_{01}$  is a precisely calculable phase-space factor. Note that it is equal to the phase-space factor of the classical light Majorana neutrino exchange mechanism. The nuclear matrix elements  $M_h^{\tilde{q}}$  depend on the hadronization mode. We denote the matrix elements for the 2N and pion modes as  $M_{2N}^{\tilde{q}}$  and  $M_{\pi}^{\tilde{q}}$ , respectively.

We derive these nuclear matrix elements up to the order of  $1/m_p$  in the non-relativistic expansion. They have the following form.

A) *The two nucleon mode* results in the full matrix element

$$M_{2N}^{\tilde{q}} = M_{AP}^{\tilde{q}} + M_{MT}^{\tilde{q}} + M_{VT}^{\tilde{q}}. \quad (23)$$

The partial nuclear matrix elements  $M_{AP}^{\tilde{q}}$ ,  $M_{MT}^{\tilde{q}}$  and  $M_{VT}^{\tilde{q}}$  have their origin in the interference of the axial-vector and pseudoscalar currents, weak-magnetic and tensor currents, and vector and tensor currents, respectively. We note that the  $M_{VT}^{\tilde{q}}$  contribution was neglected in previous studies [15].

Using the method of second quantization in the relative coordinates we obtain

$$M_{AP}^{\tilde{q}} = \langle H_{AP-GT}^{\tilde{q}}(r_{12})\sigma_{12} + H_{AP-T}^{\tilde{q}}(r_{12})S_{12} \rangle, \quad (24)$$

$$M_{MT}^{\tilde{q}} = \langle H_{MT-GT}^{\tilde{q}}(r_{12})\sigma_{12} + H_{MT-T}^{\tilde{q}}(r_{12})S_{12} \rangle, \quad (25)$$

$$M_{VT}^{\tilde{q}} = \langle H_{VT-F}^{\tilde{q}}(r_{12}) \rangle. \quad (26)$$

B) *The pion mode* nuclear matrix element obtained within the same framework is

$$M_{\pi}^{\tilde{q}} = \langle H_{\pi N-GT}^{\tilde{q}}(r_{12})\sigma_{12} + H_{\pi N-T}^{\tilde{q}}(r_{12})S_{12} \rangle. \quad (27)$$

In the above formulas we use the notations:

$$\begin{aligned} \mathbf{r}_{12} &= \mathbf{r}_1 - \mathbf{r}_2, \quad r_{12} = |\mathbf{r}_{12}|, \quad \hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{r_{12}}, \\ \sigma_{12} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\ S_{12} &= 3(\vec{\sigma}_1 \cdot \hat{\mathbf{r}}_{12})(\vec{\sigma}_2 \cdot \hat{\mathbf{r}}_{12}) - \sigma_{12}. \end{aligned} \quad (28)$$

Here,  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are the coordinates of the nucleons undergoing beta decay. The form of the matrix element  $\langle \mathcal{O}(1,2) \rangle$  within the pn-QRPA will be presented in the next section.

The neutrino-exchange potentials in Eqs. (24)-(26), (27) are defined as

$$\begin{aligned} H_{type-K}^{\tilde{q}}(r_{12}) &= \frac{2}{\pi g_A^2} R \times \\ &\int_0^\infty f_K(qr_{12}) \frac{h_{type-K}^{\tilde{q}}(q^2) q dq}{q + E_J^m - (E_{g.s.}^i + E_{g.s.}^f)/2}, \end{aligned} \quad (29)$$

where *type* - *K* stands for  $VT - F, MT - GT, MT - T, AP - GT, AP - T, \pi N - GT, \pi N - T$ . We also define  $f_{F,GT}(qr_{12}) = j_0(qr_{12})$  and  $f_T(qr_{12}) = -j_2(qr_{12})$ , where  $j_{0,2}$  are the spherical Bessel functions. In the denominator of Eq. (29)  $E_{g.s.}^i$  and  $E_{g.s.}^f$  are the ground state energies of the initial and final nuclei, respectively, while  $E_J^m$  is the energy of the intermediate nuclear states.  $R = r_0 A^{1/3}$  is the mean nuclear radius with  $r_0 = 1.1 \text{ fm}$ . The functions  $h_{type-K}^{\tilde{q}}$  in Eq. (29) can be expressed in the following form:

$$\begin{aligned} h_{VT-F}^{\tilde{q}}(\vec{q}^2) &= -g_V(\vec{q}^2) \left( T_1^{(3)}(\vec{q}^2) - 2T_2^{(3)}(\vec{q}^2) \right) \\ &\times \frac{\vec{q}^2}{2m_p m_e}, \end{aligned} \quad (30)$$

$$\begin{aligned} h_{MT-GT}^{\tilde{q}}(\vec{q}^2) &= -\frac{2}{3} (g_V(\vec{q}^2) + g_M(\vec{q}^2)) T_1^{(3)}(\vec{q}^2) \\ &\times \frac{\vec{q}^2}{2m_p m_e}, \end{aligned} \quad (31)$$

$$\begin{aligned} h_{AP-GT}^{\tilde{q}}(\vec{q}^2) &= \frac{1}{6} g_A(\vec{q}^2) F_P^{(3)}(\vec{q}^2) \\ &\times \frac{m_\pi^2}{2m_p m_e} \frac{\vec{q}^2}{\vec{q}^2 + m_\pi^2}, \end{aligned} \quad (32)$$

$$\begin{aligned} h_{MT-T}^{\tilde{q}}(\vec{q}^2) &= -\frac{1}{2} h_{MT-GT}^{\tilde{q}}(\vec{q}^2), \\ h_{AP-T}^{\tilde{q}}(\vec{q}^2) &= h_{AP-GT}^{\tilde{q}}(\vec{q}^2), \end{aligned} \quad (33)$$

TABLE II: Nuclear matrix elements (NMEs) of the squark-neutrino  $\tilde{R}_p$  SUSY mechanism of  $0\nu\beta\beta$ -decay. The NMEs of the 2N-mode are calculated for the two cases of the nucleon form factors: Quark Bag Model (QBM) and Non-Relativistic Quark Model (NRQM). The quantities  $M_{2N}$ ,  $M_\pi$  are the 2N and pion mode nuclear matrix elements averaged over small, medium and large model spaces (see the text) with their variance  $\sigma$  given in parentheses.

nucl.	QBM				NRQM				$M_\pi^{\tilde{q}}$
	$M_{VT}^{\tilde{q}}$	$M_{MT}^{\tilde{q}}$	$M_{AP}^{\tilde{q}}$	$M_{2N}^{\tilde{q}}$	$M_{VT}^{\tilde{q}}$	$M_{MT}^{\tilde{q}}$	$M_{AP}^{\tilde{q}}$	$M_{2N}^{\tilde{q}}$	
$^{76}\text{Ge}$	185.	-246.	29.6	-20.0 (33.8)	102.	-258.	31.2	-113. (25.7)	604. (74)
$^{100}\text{Mo}$	220.	-244.	33.0	22.9 (1.8)	121.	-256.	34.8	-100. (14.3)	594. (80)
$^{130}\text{Te}$	179.	-206.	28.4	6.3. (20.8)	99.2	-217.	29.8	-81.6 (17.2)	517. (32)

$$h_{\pi N-GT}^{\tilde{q}}(\vec{q}^2) = -\frac{1}{6} g_A^2(\vec{q}^2) \frac{m_\pi^4}{m_e(m_u + m_d)} \times \frac{\vec{q}^2}{(\vec{q}^2 + m_\pi^2)^2}, \quad (34)$$

$$h_{\pi N-T}^{\tilde{q}}(\vec{q}^2) = h_{\pi N-GT}^{\tilde{q}}(\vec{q}^2), \quad (35)$$

where the nucleon form factors  $g_K$  ( $K = V, A$  and  $M$ ),  $F^{(3)}$  and  $T^{(3)}$  are defined in sect. III. In the case of the pion-exchange mechanism, Eq. (34), we used the Goldberger-Treiman relation [20].

The following comment is in order. Within the considered non-relativistic approximation the NMEs of the 2N-mode, Eqs (30)-(33), depend nearly on all the nucleon form factors except for  $F_S^{(3)}$  and  $T_3^{(3)}$ . As seen from Table I these form factors are rather model dependent quantities. On the other hand the pion-mode NMEs, Eqs. (34)-(35), involve only the axial-vector form factor  $g_A$ , which is well-known from experimental measurements. Thus, the pion-mode NMEs are expected to be significantly less dependent on nucleon structure models than the NMEs of the 2N-mode.

## V. CALCULATION OF NUCLEAR MATRIX ELEMENTS

We calculate the NMEs introduced in the previous section. The partial matrix elements in Eqs. (24)-(27) are given by the expression [19]:

$$M_K = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pp'p'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \times \quad (36)$$

$$\sqrt{2\mathcal{J}+1} \left\{ \begin{matrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{matrix} \right\} \times$$

$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J} \rangle \times$$

$$\langle 0_f^+ | [c_p^\dagger \tilde{c}_{n'}]_J | J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f i | [c_p^\dagger \tilde{c}_n]_J | 0_i^+ \rangle.$$

The sum over  $J^\pi$  represents the summation over the states of the intermediate odd-odd nucleus with angular momentum  $J$ , parity  $\pi$  and energies  $E_{J^\pi}^{k_i, k_f}$ . The indices  $k_i$  and  $k_f$  represent states which are calculated from the

ground state of the initial ( $i$ ) and final ( $f$ ) nuclei, respectively. The overlap factor  $\langle J^\pi k_f | J^\pi k_i \rangle$  accounts for the difference between them. The reduced matrix elements of the one-body operators  $c_p^\dagger \tilde{c}_n$  ( $\tilde{c}_n$  denotes the time-reversed state) in (36) depend on the BCS occupation coefficients  $u_i, v_j$  and on the QRPA vectors  $X, Y$  [19].

The operators  $O_K$ , with  $K = F, GT, T$ , in (36) depend on the distance  $r_{12}$  between two initial neutrons that are transferred into two protons, the relevant spin and isospin operators as well as on the energies of the excited states  $E_{J^\pi}^{k_i, k_f}$ , albeit in practice rather weakly. Short range correlations of the two initial neutrons and the two final protons are described by the Jastrow function  $f(r_{12})$  according to Ref. [21].

We apply the pn-QRPA to calculate the  $0\nu\beta\beta$ -decay NMEs for  $^{76}\text{Ge}$ ,  $^{100}\text{Mo}$  and  $^{130}\text{Te}$ . For these nuclei strong experimental limits on the  $0\nu\beta\beta$ -decay half-life were found [see Eqs. (1) and (2)] and these nuclei are also considered as candidate sources for the next generation of the  $0\nu\beta\beta$ -decay experiments. For each of them three choices of single particle basis (minimal, intermediate and large s.p. model spaces) are considered according to Ref. [22]. The single particle energies are obtained from a spherical, Coulomb corrected Woods-Saxon potential. The Brueckner G-matrices of the Bonn potential are used as a two-body interaction. The pairing interactions of the nuclear Hamiltonian  $H$  are adjusted to fit the empirical pairing gaps. In addition, we renormalize the particle-particle and particle-hole channels of  $H$  by introducing the parameters  $g_{pp}$  and  $g_{ph}$ , respectively. In the calculation we use  $g_{ph} = 1.0$  and  $g_{pp}$  is fixed so that the known half-life of the  $2\nu\beta\beta$ -decay is correctly reproduced for each model space [23]. Recently, it was found that such a procedure makes the  $0\nu\beta\beta$ -decay NMEs essentially independent of the size of the single particle basis and the nuclear structure input [22, 23]. This procedure allowed us also to evaluate the uncertainties in the calculated NMEs.

Numerical results of the NME calculation are summarized in Table II. The presented values of the partial nuclear matrix elements  $M_{VT}^{\tilde{q}}$ ,  $M_{MT}^{\tilde{q}}$  and  $M_{AP}^{\tilde{q}}$  have been obtained for the intermediate s.p. model space: 12 levels for A=76, 16 levels for A=100, and 18 levels for A=130.

The values of NMEs for the three studied isotopes are comparable in magnitude. We have found that the considered two-nucleon short range correlations suppress the pion-mode NME by about 20%. We recall that the similar reduction occurs also for the NMEs of the “classic” light Majorana neutrino mass mechanism of the  $0\nu\beta\beta$ -decay.

As seen from Table II, the pion-mode NMEs are from 6 to 30 times larger, depending on the nucleon structure model, than the corresponding 2N-mode NMEs for all the studied nuclei. This allows us to conclude that in the squark-neutrino  $\tilde{R}_p$  SUSY mechanism of  $0\nu\beta\beta$ -decay the pion-mode of hadronization clearly dominates over the 2N-mode.

Let us point out that in the 2N-mode NMEs there is a strong cancelation between the  $M_{VT}^{\tilde{q}}$  and  $M_{MT}^{\tilde{q}}$  partial matrix elements (see Table II) originating from the tensor nucleon current (18). As a result despite the individual values of both matrix elements  $M_{VT}^{\tilde{q}}$  and  $M_{MT}^{\tilde{q}}$  are significantly larger than the pseudoscalar contribution  $M_{AP}^{\tilde{q}}$  it turns out that their cumulative effect,  $M_{VT}^{\tilde{q}} + M_{MT}^{\tilde{q}}$ , is suppressed and can be either comparable in magnitude with  $M_{AP}^{\tilde{q}}$  or larger than it, depending on the model of nucleon structure (QBM or NRQM). At this point we disagree with Ref. [15] claiming that the tensor contribution is always significantly larger than the pseudoscalar one. This disagreement can be accounted for the fact that the contribution  $M_{VT}^{\tilde{q}}$  was missed in Ref. [15] leaving the large term, analogous to our  $M_{MT}^{\tilde{q}}$ , uncompensated. As a result the tensor contribution, represented by only this term, becomes significantly larger than the pseudoscalar contribution for both QBM and NRQM models of nucleon form factors. As we have shown this is not the case when all the contributions of the tensor nucleon current (18) are properly taken into account.

In Table II the NMEs of the pion and 2N modes are given with their variances  $\sigma$  (evaluated according to Ref. [22]) due to uncertainties of nuclear structure input. In both cases these variances are within 10%. This comparatively low uncertainty was obtained mainly due to the procedure we chose for fixing the residual interaction of the nuclear Hamiltonian which includes the calculation with three model spaces [22, 23]. As to the uncertainties of the nucleon form factors, from Table II it is evident that they are very significant for the 2N-mode NMEs. For the two models of nucleon form factors considered here, the Quark Bag Model and the Non-Relativistic Quark Model, the values of the partial NMEs of the 2N-mode change up to a factor 2 while the total 2N-mode NMEs up to an order of magnitude. Fortunately, as we commented at the end of Sect. IV the dominant pion-mode contribution is free of the nucleon structure uncertainties making their impact on the resulting NMEs insignificant.

TABLE III: Upper bounds on the  $\tilde{R}_p$  SUSY parameter  $\eta_{(q)LR}^{11}$  as well as on the related products of the trilinear  $\tilde{R}_p$ -couplings  $\lambda'_{11k}\lambda'_{1k1}$  ( $k=1,2,3$ ) for  $\Lambda_{SUSY} = 100$  GeV (see scaling law in Eq. (37)) deduced from the current lower bounds on the half-life of  $0\nu\beta\beta$ -decay for  $^{76}Ge$ ,  $^{100}Mo$  and  $^{130}Te$ .

nucl.	$T_{1/2}^{0\nu-exp}$ [Ref.]	$\eta_{(q)LR}^{11}$	$\lambda'_{111}\lambda'_{111}$	$\lambda'_{112}\lambda'_{121}$	$\lambda'_{113}\lambda'_{131}$
	(years)				
$^{76}Ge$	$\geq 1.9 \cdot 10^{25}$ [2]	$4.3 \cdot 10^{-9}$	$7.7 \cdot 10^{-6}$	$4.0 \cdot 10^{-7}$	$1.7 \cdot 10^{-8}$
$^{100}Mo$	$\geq 5.8 \cdot 10^{23}$ [4]	$9.2 \cdot 10^{-9}$	$1.7 \cdot 10^{-5}$	$8.7 \cdot 10^{-7}$	$3.6 \cdot 10^{-8}$
$^{130}Te$	$\geq 3.0 \cdot 10^{24}$ [5]	$4.7 \cdot 10^{-9}$	$8.5 \cdot 10^{-6}$	$4.5 \cdot 10^{-7}$	$1.9 \cdot 10^{-8}$

## VI. DISCUSSION AND CONCLUSIONS

With the values of the nuclear matrix elements of the dominant pion-mode contribution given in Table II we derived the experimental upper limits on the LNV parameter  $\eta_{(q)LR}^{11}$  introduced in Eq. (7). Then, using Eq. (8), we extracted upper limits on the products of the trilinear  $\tilde{R}_p$ -couplings  $\lambda'_{11k}\lambda'_{1k1}$  ( $k=1,2,3$ ). These limits, presented in Table III, have been derived under the conventional simplifying assumptions. We assumed all the squark masses and the trilinear soft SUSY breaking parameters  $A_d$  to be approximately equal to a common SUSY breaking scale  $\Lambda_{SUSY}$ . Thus we approximately write

$$\lambda'_{11k}\lambda'_{1k1} \leq \epsilon_k \frac{1}{\sqrt{T_{1/2}^{0\nu-exp} G_{01}}} \frac{1}{M_{\pi}^{\tilde{q}}} \left( \frac{\Lambda_{SUSY}}{100\text{GeV}} \right)^3 \quad (37)$$

with  $\epsilon_k = (1.8 \times 10^3; 94.2; 3.9)$  calculated for the current quark masses  $m_d = 9$  MeV,  $m_s = 175$  MeV and  $m_b = 4.2$  GeV respectively. In the above formula  $T_{1/2}^{0\nu-exp}$  is the experimental lower bound on the half-life of a certain nucleus. In Table III we show the upper limits on the  $\tilde{R}_p$  SUSY parameters, extracted from the existing experimental lower bounds on the half-lives of the three nuclei  $^{76}Ge$ ,  $^{100}Mo$  and  $^{130}Te$ . Our limit for  $^{76}Ge$  is about an order of magnitude better than the limit previously obtained in Ref. [14, 15] on the basis of the 2N-mode of hadronization taking into account only the pseudoscalar current contribution,  $M_{AP}^{\tilde{q}}$ .

In conclusion, we analyzed  $0\nu\beta\beta$ -decay of several nuclei induced by the LNV effective operators originating from the  $\tilde{R}_p$  SUSY trilinear interactions involving squark and neutrino exchange. We focussed on the hadronization prescription of the quark-level operators and analyzed both the conventional 2N-mode and the pion-mode of hadronization. We have shown that the pion-mode absolutely dominates over the 2N-mode. Previously, in Refs. [9] it was demonstrated that the pion-mode dominates over the 2N-mode in the case of the short-range  $\tilde{R}_p$  SUSY mechanism. Thus, with the result of the present

paper we conclude that all the mechanisms based on the trilinear  $\mathcal{R}_p$  SUSY interactions dominantly contribute to  $0\nu\beta\beta$ -decay via the pion-mode of hadronization.

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